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COMMUNICATION AND TRANSPORTATION NETWORKS RELIABILITY USING ROUTING MODELS

by

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ABSTRACT

A new technology for finding a good measure of reliability in flow networks is introduced. It consists, in its general form, in choosing a good parameter of performance Z, finding bounds on the expected value of Z by partially enumerating the m most probable states of the system and, within every state, establishing and solving a routing program. Ways to incorporate this methodology for specific flow networks are shown for every one of the following systems: communications, electric power, vehicular, bus, and air transportation.

Key words : reliability; flow networks; communication systems; electric power systems; transportation systems.

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RÉSUMÉ

Nous introduisons une nouvelle technologie pour trouver une bonne mesure de fiabilité dans des réseaux de flot. Il s'agit de choisir un bon paramètre de performance Z, d'évaluer l'espérance de Z en énumérant les m états les plus probables du système et, finalement, de résoudre un programme de routing pour chacun de ces m états. Nous montront aussi comment incorporer cette méthodologie pour des réseaux spécifiques : communications, électricité, transport routier, aérien et urbain.

Mots-clés : fiabilité; réseaux de flot; système de communications; réseaux de puissance; réseaux de transport.

1. INTRODUCTION

This paper aims at introducing a procedure to find a measure of reliability that could effectively evaluate the performance of any kind of network flow. We will specifically touch on the following types of networks: communication, electric power, vehicular, bus, and air transportation.

Traditionally, two approaches have been taken to evaluate network reliability:

1 - Consider the network as a pure graph and, knowing node and arc availability, evaluate some measure of connectivity as the reliability of the network. This type of approach considered the reliability as a "static" measure since failed elements were not supposed to be repaired and their probability of failure was a known, fixed value.

2 - Take into account that the purpose of a network is to carry flow from origin to destination nodes to satisfy a certain demand. The reliability measures in this approach were always related to the probabilities of coping or not with specific levels of demand. Most of the time this approach considers the reliability as a "dynamic" measure to be found by taking into account the possibility of repairs and time-variable element availability.

Many of the measures of connectivity proposed in the first approach have been proven or suspected to give rise to NP-hard problems [1]. Moreover, probabilities of connectivity in most real networks are usually high; therefore, taking connectivity as a measure of reliability can lead to an overestimation of the network performance. Furthermore, the first approach does not take into account the flow nature of the network, and related parameters such as arc capacity and demand are completely left out of consideration.

The second approach is typical of physical networks such as electric power systems [2]. The clear advantage of this approach is the fact that the flow nature of the network as well as repairs and time variant reliability are taken into account. Even though this approach considers a "dynamic" measure of reliability ("dynamic" in the sense that repair and failure times are taken into account), the management of the system is a "static" one since no action-taking procedure is considered regarding routing decisions.

We propose a measure of "static" reliability (failure duration is not considered) that will take into account several models for routing decision procedures.

Before entering into the details of every kind of network flow, we outline the general methodology that will later be applied to specific networks. Once within a specific system, we discuss the following points: the mathematical way of describing the system and the measure of reliability to be considered, how failures in the physical system have to be managed so as to use the general methodology to be shown in this paper, and finally, available routing models.

2. GENERAL METHODOLOGY

The procedure for finding a good measure of performance in any of the network flow problems to be presented is summarized below:

1 - choose a parameter Z such that the expected value of Z is the desired measure of reliability;

2 - considering the kinds of failure a network can have, partially enumerate the m most probable states of the network;

3 - for each one of the m states enumerated, find the routing and the value Z for that specific scenario;

4 - once Z is known for each state of the system, evaluate the bounds on the expected value of Z by means of the Li and Silvester formulas [3] which will be shown in section 2.2.

2.1 Choice of Measure

First of all, we need to find a measure of performance that will consider the flow nature of the network. Two different groups of measures are immediately outlined, those which evaluate reliability from the point of view of the user and those which evaluate reliability from the manager's point of view.

1 - **quality of service measures** (user point of view reliability). A suitable quality measure could be related to the number and importance of customers served (this will be applied in telecommunications and electric systems). In transportation, a quality of service measure is always related to time, for instance, mean time for a passenger to reach his destination or mean time of delay in the event of failure.

2 - **economic measures** (manager's point of view reliability). Failures can always be quantified in terms of money. Reliability can then be measured by loss of revenue in the case of failure.

2.2 Partial Enumeration

Let us define a state of the system as the state of each system component. If these components can only be failed or operating, there will be 2^n possible states (n being the number of components of the system). It is possible to use the methodology to be described later for a multi-state system. If all n devices of the system can have s operating states, there will be s^n possible states. If each device i has a particular number s(i) of states associated to it, the number of possible states is $\prod_{i=1}^{n} s(i)$. For the sake of simplicity, we refer to two-state devices: then in case a component can have several suitable and unsuitable levels of operation we consider a "failed state" as the union of all unsuitable levels and, consequently, the "operating state" will be the union of the suitable levels.

If S_k represents the state of the system and Z the measure of reliability to be evaluated, then the upper and lower bounds on the expected measure of performance can be obtained by the formulas given in [3].

$$E[Z]_{up} = \sum_{k=1}^{m} P(S_k)\, Z(S_k) + \left(1 - \sum_{k=1}^{m} P(S_k)\right) Z(S_{2^n}) \qquad (1)$$

$$E[Z]_{low} = \sum_{k=1}^{m} P(S_k)\, Z(S_k) + \left(1 - \sum_{k=1}^{m} P(S_k)\right) Z(S_1) \quad .$$

In (1) $P(S_k)$ represents the probability that the system is in the state S_k, $Z(S_1)$ is the measure of performance when there are no failures

in the system and $Z(S_{2^n})$ is the measure considered when every component in the system is in its failed state. m is the number of most probable states enumerated.

2.3 Optimal Routing

A general network can be formalized by a graph G(N,A) where N represents the set of nodes and A the set of arcs. In a network, flows usually model some physical flow quantity such as flow of persons, electric power, fluid, or money. Nodes constitute points of origin, destination or transshipment of commodities. Arcs are direct paths between nodes.

Most physical network flow problems have to be modelled as multicommodity networks, that is, networks where the flow to be transported is of differing nature, usually associated with the pair of origin and destination nodes of the flow (O/D pair).

We now present the notation necessary for constructing an optimal routing model for a multicommodity network. In the event that a single commodity network is considered, the formulation is still valid; it is only necessary to change the cardinality of the commodity set.

Notation:

X_{ij}^c : flow of commodity c flowing from node i to node j.

C_{ij} : capacity of the (i,j) arc.

d^c : demand for commodity c.

$\Gamma(i)$: set of arcs whose initial extremity is node i.

$\Gamma^-(i)$: set of arcs whose terminal extremity is node i.

N : arc set.

N_c : commodity set.

s^c : origin node for commodity c.

t^c : destination node for commodity c.

When the routing procedure is a global optimization procedure taken from the manager's point of view it can be modelled by the following multicommodity problem:

s.t. $$\min \; L(x_{ij}^c) \tag{2}$$

conservation of flow:

$$\sum_{(i,j)\in\Gamma(i)} x_{ij}^c - \sum_{(j,i)\in\Gamma^-(i)} x_{ji}^c = \begin{cases} d^c & \text{if } i=s^c \\ -d^c & \text{if } i=t^c \\ 0 & \text{otherwise} \end{cases} \quad \begin{array}{l} \text{, for all } c\in N_c \\ \text{for all } (i,j)\in N \end{array} \tag{3}$$

capacity:

$$\sum_{c\in N_c} x_{ij}^c \leq C_{ij}, \quad \text{for all } (i,j)\in N \tag{4}$$

nonnegativity:

$$x_{ij}^c \geq 0, \quad \begin{array}{l} \text{for all } (i,j)\in N, \\ \text{for all } c\in N_c \end{array} \tag{5}$$

In (2), $L(x_{ij}^c)$ represents a linear function to be minimized which can simply be a dummy one, a valid function evaluating costs or minimum paths in the network, or even the value of the measure of reliability for the state considered.

The multicommodity problem with capacity constraints is very difficult to solve. Traditionally, the capacity constraints have been dropped and replaced by an artificial penalty function to be added to the objective function. In several of the routing applications we will review, the penalty function is not an artificial one, but a carefully studied curve that either corresponds to congestion delay or reflects the stochastic character of the demand while avoiding the capacity of the arcs to be surpassed. The problem can then be reformulated as:

$$\min \; L(X_{ij}^c) + P(X_{ij}^c)$$

s.t.

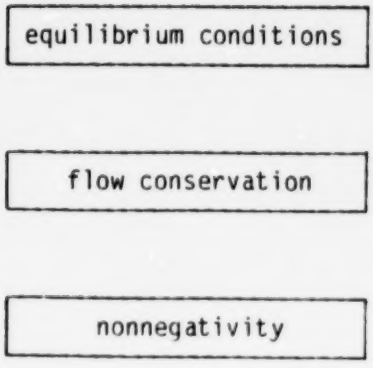
$$\boxed{\text{conservation of flow}} \tag{6}$$

$$\boxed{\text{nonnegativity}}$$

where $P(X_{ij}^c)$ is the aforementioned penalty function. Existing algorithms can solve, in reasonable time, large scale network problems formulated as (6).

The routing formulations shown up to now correspond to a global optimization model of the network. This kind of model considers that the network is managed by a sort of centralized decision maker who routes the flows in such a way as to optimize the global performance of the network. Global optimization is then a good approach to use in networks where no customers or users are involved in the outcome of the routing such as in electric power systems.

When users can decide the route to take, a decentralized behavior of the system has to be considered and the notion of "user optimization" has to be taken into account. A user optimization model can be a simple description of the equilibrium conditions of the system in order for the user to optimize a specific parameter. In general, a user optimization model can be formulated as a set of nonlinear equations as follows:



(7)

Some user optimization models are derived from the Kuhn-Tucker conditions applied to a network problem. For those models the possibility of solution by solving the corresponding network optimization problem exists. Otherwise, large scale user optimization formulations can be solved by one of the following methods: Newton Raphson, Gauss-Seidel, or variational inequalities.

3. MEASURES OF RELIABILITY ON SPECIFIC NETWORKS

3.1 Communications

The System

In a communication network, switching stations correspond to nodes and communication links to arcs. Flow in telephone and data networks can be measured in a unit of traffic called the Erlang (or CCS in the American system). Each kind of call or message going from an origin to a destination node constitutes a commodity. Different types of services such as telephone calls and computer data in public or private networks give rise to different commodities as well. The system is a multicommodity one having as many types of flows as origin-destination pairs of nodes.

A good measure of reliability for this kind of network has been proposed in [4]; it is the expected lost call traffic in the system. In order to find the measure, a methodology similar to the general one shown in section 2 was developed in [4].

Failures

In communication networks, both nodes and arcs can fail. If n is the number of arcs and s the number of nodes, there will be 2^{n+s} states to be enumerated. A partial enumeration procedure to find the m most probable states should not discriminate between the kinds of devices which produce a failed state. Therefore, switching stations and communication links can be treated as simple devices while finding the m most probable states. On the other hand, once placed in the routing subproblem, a distinction has to be made regarding the failure of arcs and nodes. In fact, when a node is failed some or all the arcs adjacent to it fail as well. We have then considered that if a state admits the failure of some nodes the corresponding routing will take place in a subgraph where the failed arcs as well as the failed nodes and their adjacent arcs will be removed.

Another problem regarding failures arises in communication networks. Usually the communication company works with a logical network in mind and failures are referred to that kind of network. Unfortunately, since most of the time, different logical paths share the same physical equipment, failures in the logical network can be dependent. The algorithm of partial enumeration considers that device failures are independent. In order to overcome this difficulty, we can choose to work on the physical network where failures are independent, and translate the necessary results into the logical one.

Routing

Several routing models of the centralized optimization kind were proposed in [4]. The basic idea was to use the nonlinear formulation of a multicommodity model given by (6). The penalty function $P(X_{ij}^c)$ chosen to avoid the capacity of arcs to be surpassed was the expected lost call traffic in an arc: $\sum_c \sum_{(i,j)} X_{ij}^c \, B(\sum_c \sum_{(i,j)} X_{ij}^c, C_{ij})$ where the function B represents the Erlang loss formula. The function $L(X_{ij}^c)$ shown in the objective of (6) was represented in [4] by a term to favor the shortest path as well as a measure of the weighted lost traffic for every origin-destination.

The authors of [4] also consider the possibility of defining a single commodity as all of the traffic flowing from the same origin node. Consequent models are then developed.

We now show another possibility of routing related to the notion of user optimization already mentioned in section 2.3.

The reason we are interested in these kinds of models is that they are closer to reality than global optimization ones. In fact, in telecommunications, the idea of calls being placed in an optimal way all over the

network does not reflect reality. A more realistic approach is to consider a decentralized decision center where decisions are made without optimization by using a priority route. In telecommunications, calls have to be assigned to a specific route; those that cannot find a line in this route, will pass to a second route and so on until all possible routes have been tried.

Our decentralized approach models an origin-destination behavior shown in figure 1.

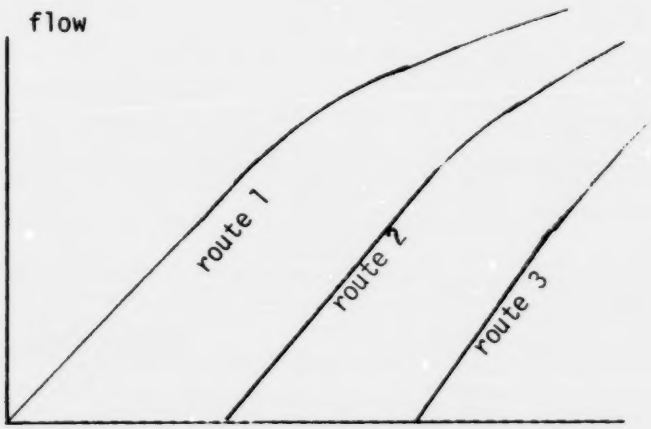


Figure 1:

All demands try to pass through route 1. The excess flow of route 1 passes through route 2. The excess flow of route 3 is lost. It is interesting to note that even though there is a strict priority in the choice of routes, Figure 1 shows that a route continues to accept calls even though the next priority route is operational. The explanation depends on the stochastic behavior of the flow.

Let

X_q : average traffic trying to pass through route q.

h_q : average traffic flowing through route q.

$P_q(X_q)$: expected lost call traffic of route q.

D : demand for an O/D pair.

Then,

$$X_q = h_q + P_q(X_q), \quad \text{for all } q=1,\ldots,4 \tag{8}$$

In (8) we have introduced a dummy route (the fourth one) which receives the total traffic lost for the O/D pair.

Therefore

$$\begin{aligned} X_1 &= D \\ X_2 &= P_1(X_1) \\ X_3 &= P_3(X_3) \\ X_4 &= P_4(X_4) \end{aligned} \tag{9}$$

(9) correponds to a set of equations that model the behavior for one O/D pair. Where all origin-destination pairs are considered, (9) becomes a more complex equation system since several O/D routes can have arcs in common. Therefore, the excess flow in an O/D route depends not only on the flow from this very route, but from all flows of the different O/D that use the common arcs in the route. Then we can rewrite (9) as:

$$X_{i+1} = P_i(X_i, x), \quad i=1,2,\ldots,3 \tag{10}$$

where x is a function englobing the flow variables in the common arcs. The equation system obtained by applying (10) for all O/D pairs can be solved by numerical methods such as Gauss-Seidel or Newton-Raphson.

Regarding an expression for $P_i(X_i, x)$, we do not derive it here. We refer the interested reader to a forthcoming publication "Routing Models in Telecommunication Systems" by the same authors.

3.2 Electric Power Systems

The System

A complete power system is composed of generation, transmission, and distribution facilities. The aim of the system is to meet the power requirements of the changing load.

Traditionally, power system reliability was studied in terms of indices that showed the ability of the system to cope with the demand [2]. In order to determine the reliability indices, the entire system was not considered; the system was rather decomposed into suitable subsystems and the ability to satisfy customer load requirements was studied within each subsystem.

In our approach we will consider the system as a whole. The network is modelled as a graph G(N,A) where N, the set of nodes, represents generating stations, switching stations, or load bars, and A, the set of arcs, represents the transmission lines.

The commodity to be transmitted is electric power measured in MVA (Mega Volt-Ampers) and constitutes a complex quantity.

Measures of user as well as operating company point of view reliability can be chosen. In either case the measure will be expected values or weighted expected values calculated by carrying out the partial enumeration procedure and the routing for every state.

A measure of service quality could be the expected number of times a specific customer or region is left without service or the weighted expected number of customers shut off in the study period.

From the company's point of view, economic measures such as expected generation costs or expected loss of revenue due to failures may be taken into account.

Failures

Many different kinds of failures can occur in a power system. Regarding the failure of generating stations, generating units usually have different capacity levels, and partial as well as total failures can be considered. It is up to the power company to set the suitable levels of generation so as to define the availability of the unit as the probability of being in one of the suitable states.

Failures of the transmission lines can be more difficult to deal with. They account for 50% of the total possible failures in the system [5], so it is important to identify them before making any assumptions. The difficulty in dealing with transmission line failures is based on their diversity. In fact, nearly all power systems operate in a triphasic way, therefore line failures can be monophasic (line to earth), biphasic (two lines or two lines to earth), or triphasic (three lines to earth). Each kind of failure gives rise to a different scenario of overcurrent distribution that can considerably complicate the routing flow in the network.

Monophasic failures (line to earth) are the most common ones; they account for 85% of all line failures [5]. The response of the system to each different failure may differ depending on system configuration and how the engineer designed the protection system to act.

Since the aim of this work is to give a general framework for future specialists to study their respective system reliability, we choose simplicity when modelling failures in transmission lines. Therefore, the most common failures should be considered, and we assume that after a failure only the affected line is put out of service; transitory effects will not be taken into account.

Routing

Traditionally, power engineers have routed power flows using load flow studies. This consists in establishing a set of flow conservation equations (the flow being the complex power) and solving it by means of a numerical method such as Newton-Raphson or Gauss-Seidel. Capacity as well as load levels are pre-specified. The objective of the load flow study is to establish voltage levels in every node (so as to calculate power flow). Once this is done constraints on the bounds of bus voltage and power capacity must be verified. In case the constraints are not verified, it is up to the knowledge and ability of the operator to re-arrange parameters and rerun the program. There are, therefore, many possible solutions to load studies.

The obvious goal for an operations researcher is to seek an optimal solution. The way to find an optimum set of flows in the network is not obvious at all. Many efforts have been made to incorporate load flow studies into an optimal routing program usually known as optimum economic dispatch or the general optimum operational problem [6]. However, most of the proposed formulations have to make some kind of approximation due to the complex nature of the flow.

3.3 Vehicular Traffic

The System

A vehicular traffic system is composed of vehicles (cars) and routes. Vehicles travel from origin to destination nodes using different roads of the network. The system can then be modeled as a flow network where arcs are roads, nodes are origin-destinations or interconnection points and vehicles constitute the flow. The system is a multicommodity

network whereby one has to distinguish between vehicles of different origin-destination nodes. A commodity can then be defined as all vehicles travelling from the same origin point to the same destination.

Reliability has not, to our knowledge, been studied in vehicular traffic. We propose to use the general framework of evaluating a measure of reliability in network flows to find the following index of vehicular traffic reliability given from the user's point of view:

- Expected value of the total transportation time of the system. This measure may also be weighted for every different kind of O/D flow.

Failures

Failures in this kind of system represent accidents on the roads. Once more we have to consider the total deterioration of the road (arc) where the accident takes place in order to have only two possible states of operation. In this system, nodes cannot fail, therefore if n is the number of arcs in the network, there are 2^n possible states.

We have to make an assumption about the dependency of failures; we suppose that accidents happen at random and there is no dependency between them. Finally, once an accident has taken place on a road this must be considered as a shut down, that is, the event of another accident happening in the failed road does not affect the outcome of the measure of "static" reliability (we do not consider the lapse of time nor the possibility of repairing the failure).

Routing

The prime objective of vehicular traffic systems modelling is to predict the behavior of the flow in order to evaluate different features of the system. We have already seen in section 2.3 that there are two ways of optimizing the routing in a flow network. One is by "global optimization", the other is by "user optimization".

The measure of reliability can be considered with a global multicommodity model in mind. The equation of the model can be similar to those shown in section 2.3 (equations 2 to 6) and the objective function $L(X_{ij}^c)$ is:

$$L(X_{ij}^c) = \sum_{(i,j)} \sum_{c \in N_c} t_{ij} X_{ij}^c \qquad (10)$$

where

t_{ij} = time necessary per unit of flow to go from node i to node j.

In this case, the outcome of the objective function for each routing subproblem is the measure of reliability for the given state $Z(S_k)$ to be used in the evaluation of equation 1 (bounds on the expected value of reliability).

The measure of reliability can be found with a user optimization routing formulation, that is, each user chooses his route from the origin to the destination point in order to minimize his own transportation time. The idea is not to optimize flows in the system with a specific global objective in mind but to establish the equilibrium equations for the network and observe the behavior of the flow.

Routing models based on the user optimization principle in transportation networks are shown in [7]. Large scale problems can be solved by using an adaptation of the Frank-Wolfe algorithm.

3.4 Air Transportation

The System

The air system can be represented by a space-time network where nodes constitute the intersection of a place and a time of arrival or departure from that place. Arcs represent segments of flights between cities.

The flows in the network are the passengers and the kind of flow will be determined by the origin-destination points associated with each passenger. Before getting into any other details of the system, we introduce some suitable definitions (extracted from [8]).

Flight : service description. A flight is defined by a plane, an itinerary (Montreal-Rio-Buenos Aires), departure and arrival time.

Arc or flight segment : nonstop segment of a flight (Montreal-Rio).

Route : path from an origin to a destination used by the passengers.

A good measure of reliability from the company's point of view could be the expected number of excess passengers in the study period. This measure could be weighted according to the importance of the market and could be translated into expected loss of revenues in the event of failure.

If we assume that all passengers of a specific market have a choice (in a pre-specified priority) of routes, a measure of reliability from the passenger point of view could be the expected priority number of the route finally used for the passenger. Another measure could be related to the actual time of travel in the event of failure.

Failures

A failure in an air transporation network can be characterized by the absence of one or more arcs in the network. For instance, a delay due to bad weather or unscheduled airplane repairs can be considered as failure. When a failure occurs before the beginning of the flight, not one but all arcs belonging to that flight must be removed from the network and

eventually placed later in time. If the failure takes place in the middle of a flight, all arcs from the specific stop where the failure occurred to the end of the flight should be removed. Conceptually, the number of possible states of failure for this kind of definition is $\prod_{F=1}^{f} n(F)$, where f is the number of flights, F represents a specific flight, and n(F) is the number of nodes (stops) in the flight F (including departure and arrival nodes). In reality, the number of states is smaller than that. In fact, the same airplane is normally used for several flights and if a flight is removed or re-scheduled, all consecutive flights to be assigned to the same airplane have to be removed too. Instead of increasing the number of possible states, this decreases them. The number of states is $\prod_{A=1}^{a} n(A)$, where a is the number of available airplanes, A represents a specific plane, and n(A) is the number of nodes in the union of flights assigned to airplane A during the study period.

Routing

Soumis [9] has developed four routing models in air transportation: two correspond to the global optimization notion, and the other two model the system from the user's point of view.

In [9], the model P1 minimizes the loss of revenue due to the excess of passengers and the use of nonattractive routes. Model P2 introduces attraction coefficients per route; the objective of model P2 is composed of a loss of revenue term and a second term associated with passenger insatisfaction for not being able to travel on the route of their preference. P3 is a user optimization model; passenger behavior is modeled for each origin-destination and it is considered that each passenger chooses his itinerary following his preference and accepts pre-determined alternative routes if his first choice is not available. P4 is a user optimization model similar to P3 but the choice of not travelling if the suitable routes are not available is taken into consideration as well as an elastic demand.

3.5 Bus Urban Transportation

The System

The system is composed of lines and bus and metro stations. Vehicle (bus and train) capacity as well as frequency of each line must be known. In this type of network, passengers constitute the flow.

The network is then a static one pre-determined by line routes where nodes are physical points (stops or stations). The dynamic character of the network (passengers departing from a destination at a specific hour) is then taken into account by the frequency and delay on the lines rather than by representing a space-time network (such as in air transporation). An index of reliability must reflect the passenger perception of the system. A good measure could be the expected travelling time in the event of failure, weighted or not for every origin-destination. Another possible measure could be the expected delay for every origin-destination pair based on the mean time of travelling when no failures are present.

Failures

Physically, failures represent delays due to accidents or unexpected congestion on the lines. In our model, failures should be represented by a reduction on line frequency, therefore, each possible state for a line corresponds to a possible frequency. If F(L) denotes the number of levels of frequency for line L and ℓ is the number of lines in the system, then $\prod_{L=1}^{\ell} F(L)$ is the maximum number of states that the system can have.

Routing

The latest studies in urban transporation model the network with the user optimization criteria in mind. This corresponds to reality since passengers choose their routes from origin to destination nodes trying to

minimize their own travelling time and not cooperating with other passengers so as to make global travelling time minimum. The term "routing" is then applied to the assignment of passengers to specific lines. An excellent study on the subject has been prepared by Gendreau [10].

4. CONCLUSIONS AND FURTHER WORK

This paper has established a general framework for calculating a reliability measure in different systems whose common point is to carry flow.

The methodology employed gives good bounds on the expected measure of reliability without having to account for all possible states of the system. In fact, since devices are generally reliable and multiple failures are rare, a little common sense can reduce considerably the scale of the system.

Regarding the routing of flow in multicommodity systems, the proposed nonlinear formulations allow large scale systems to be solved in reasonable time which cannot be done when the multicommodity problem is formulated with explicit arc capacity constraints.

The union of all the features described above constitutes a new technology for reliability evaluation that accurately assess the performance of the network without being too expensive to obtain. In communications, we are currently working on an efficient algorithm to assess the proposed measure of reliability in a real network. In electric power systems, a new measure of global reliability can be introduced following the proposed procedure. In transportation systems, very little has been done to evaluate reliability. Therefore, this new technology opens the way to research in each of the fields described.

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